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Vehicle Obstacle Avoidance Path Planning Based on Gauss Pseudospectral Method

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Abstract: For the problem of vehicle obstacle avoidance control, a steering obstacle avoidance method based on Gauss pseudospectral method is designed. In this paper, the kinematics and dynamics models of vehicles are established as dynamic constraints, combining with state constraints and path constraint, the obstacle avoidance behavior of intelligent vehicles is summarized as a continuous optimal control problem which takes obstacle avoidance time as the optimal objective. In the research of solving the optimal control problem, Gauss pseudospectral method is adopted which utilizes numerical approximation techniques to transform the continuous optimal control problem into a discrete nonlinear programming problem with series of equality and inequality constraints. The trajectories of control variable and state variable are obtained through solving this nonlinear programming problems numerically. The effectiveness and control performance of this method are illustrated by simulation.

Key Words: vehicle obstacle avoidance, path planning, Gauss pseudospectral method

1 Introduction

With the development of automotive intelligence, obstacle avoidance control, as an essential part of autonomous driving technology, has become a hot research topic in the automotive industry. Path planning for obstacle avoidance has been the major focus of academic attention and extensive research.

In the existing vehicle obstacle avoidance path planning research, artificial potential field and virtual force field are two traditional methods. The artificial potential field algorithm is applied to the local path planning of intelligent vehicles in [1]. A dynamic obstacle and a virtual gravitational potential field are constructed, it can realize small angle and large radius obstacle avoidance of intelligent vehicles. The planned path is smooth and safe, however, this algorithm has the problem of falling into local minimum. Virtual force field method is a kind of obstacle avoidance algorithm combining grid method and artificial potential field method. It has great advantages in uncertain dynamic environment, but it is difficult to deal with path planning in complex traffic scenarios [2]. In addition, intelligent optimization algorithms have been developed. Literature [3] proposes a continuous curvature RRT algorithm, which combines environmental constraints and vehicle constraints in the RRT framework. However, RRT has the problem of algorithm speed, and the search efficiency is difficult to guarantee. Literature [4] uses fuzzy logic and genetic algorithm to construct an intelligent vehicle obstacle avoidance path planning algorithm. Since the fuzzy logic algorithm is mainly based on human driving experience, it has the disadvantage of poor flexibility.

At present, most of the researches focus on the direct algorithm of path planning. Among them, the optimal trajectory planning based on pseudospectral method has developed rapidly in recent years and is widely used in complex optimal control problem. It can consider a variety of complex constraints in the control problem and solve the optimal trajectory by discretization.

For the problem of vehicle obstacle avoidance control in this paper, Gauss pseudospectral method is proposed to solve the path planning of vehicles, and simulation verification is carried out in the software of Matlab.

The second part describes the requirements of vehicle obstacle avoidance control, the kinematics model and three degree of freedom vehicle dynamics model are established in the third part, the fourth part establishes the optimal control problem of vehicle obstacle avoidance considering constraints, which takes obstacle avoidance time as the optimal objective, the optimal control problem is discretized by Gauss pseudospectral method and transformed into a nonlinear programming problem, the fifth part illustrates the performance of this method through simulation and the sixth part concludes the paper.

2 Problem Setup

The traffic environment is complex and changeable, vehicles may encounter various dynamic and static obstacles during driving, there are usually two kinds of vehicle obstacle avoidance strategies. One is to reduce the speed by pressing brake pedal to stop the vehicle before reaching the obstacle and avoid collision. The other is to avoid collisions by adjusting the steering wheel to change lanes. Considering the traffic efficiency and other aspects, the latter is usually adopted when conditions permit. This paper assumes that the vehicle sensing system can provide the required information for the vehicle itself, and the perception system

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can provide environmental information. As shown in figure 1, vehicle A detects the obstacle C driving at a low speed in front of it and takes the way of changing the lane to avoid collision. During this operation, the influence of other vehicles B in the traffic environment also need to be considered.

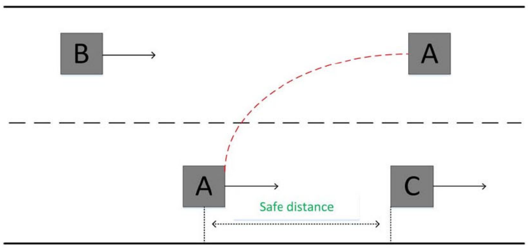

Fig. 1: Working condition diagram

In the above obstacle avoidance control problem, the start time of obstacle avoidance operation is known. At this point the vehicle drives in a certain stable state, the state of the vehicle at the initial moment is known. After changing lanes to the left, the vehicle continue to drive in a steady state, so the expected vehicle status at the terminal moment is known. However, the time that vehicle A reaches the target state is unknown. In summary, the obstacle avoidance control problem of intelligent vehicles can be summarized as a type of constrained optimization problem where the initial time and state are known, the terminal target state is known, and the terminal time is to be optimized.

3 Vehicle Model

3.1 Kinematic Model

In order to describe the kinematic characteristics of the vehicle, assume that the vehicle is a rigid body and establish kinematic equation to describe the change of the vehicle's position in the geodetic coordinate system, as shown in the Fig. 2. XOY is the global coordinate system, its kinematic model is shown in equation (1).

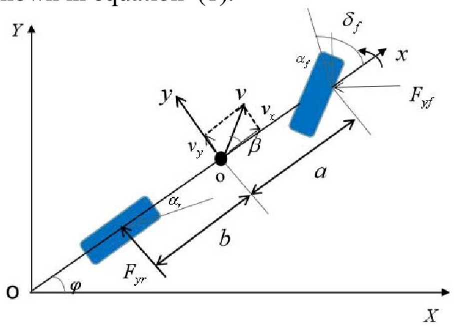

Fig. 2: vehicle model

$$\begin{aligned} \dot{x} &= v\cos(\varphi+\beta) \\ \dot{y} &= v\sin(\varphi+\beta) \\ \dot{\varphi} &= v\frac{\tan\delta_f}{(a+b)}\cos\beta \end{aligned} \tag{1}$$

where x, y are the longitudinal and lateral displacements, φ is the heading angle of the vehicle, v is the vehicle center of mass velocity, β is the sideslip angle, δ_f is the steering angles of front wheels, a is the distance from center of gravity (COG) to front axle and b is the distance from COG to rear axle.

3.2 Dynamical Model

Generally, the two-degree-of-freedom bicycle model assumes that the vehicle's longitudinal speed is constant. In order to reflect the impact of speed on path planning of intelligent vehicles, this paper establishes a three-degree of freedom vehicle model, as shown in Fig. 2. Ignoring air resistance and tire longitudinal forces, the model contains three state variables, longitudinal speed, lateral speed and yaw velocity[5].

$$\begin{aligned} \dot{v}_x &= v_y w - \frac{F_{sf}\sin\delta_f}{m} \\ \dot{v}_y &= \frac{F_{sf}\cos\delta_f + F_{sf}}{m} - v_x w \\ \dot{w} &= \frac{aF_{sf}\cos\delta_f - bF_{sr}}{I_z} \end{aligned} \tag{2}$$

where v_x is the longitudinal velocity, v_y is the lateral velocity, ω is the yaw velocity, m is the vehicle mass, I_z is the yaw inertia moment about z axes, F_{sf} and F_{sr} are the front and rear wheel lateral forces respectively.

Assume that vehicle drives normally and stably without entering the extreme conditions, the lateral tire force of the vehicle does not reach saturation, the front and rear lateral forces can be described as:

$$\begin{aligned} F_{sf} &= 2C_f\alpha_f \\ F_{sr} &= 2C_r\alpha_r \end{aligned} \tag{3}$$

where C_f, C_r are the lateral stiffness of front wheel and rear wheel, α_f and α_r are the side slip angle of front and rear wheels.

According to the rules of the coordinate system, the front slip angle α_f and the rear slip angle α_r can be approximately described as (4)[6].

$$\begin{aligned} \alpha_f &= \beta + \frac{a\omega}{v_x} - \delta_f \\ \alpha_r &= \beta - \frac{b\omega}{v_x} \end{aligned} \tag{4}$$

Equation (3) and (4) are taken into equation (2) to obtain the final three-degree of freedom vehicle model (5).

$$\begin{aligned} \dot{v}_x &= v_y\omega - 2C_f\sin\delta_f\left(\frac{v_x\beta + a\omega - v_x\delta_f}{mv_x}\right) \\ \dot{v}_y &= 2C_f\cos\delta_f\left(\frac{\beta v_x + a\omega - v_x\delta_f}{mv_x}\right) + 2C_r\left(\frac{\beta v_x - b\omega}{mv_x}\right) - v_x\omega \\ \dot{\omega} &= 2aC_f\cos\delta_f\left(\frac{\beta v_x + a\omega - v_x\delta_f}{I_Z v_x}\right) - 2b\left(\frac{\beta v_x - b\omega}{I_z v_x}\right) \end{aligned} \tag{5}$$

Select state variables $x = \left[x, y, \varphi, v_x, v_y, \omega, \beta, \delta_f\right]^T \in X$, because the speed and steering angle of the vehicle cannot be changed at the same time, it is discontinuous to use v_x and δ_f as control variables[7], therefore, select the control variables $u = \left[\dot{v}_x, \dot{\delta}_f\right]^T$. Combined with the vehicle kinematics and dynamics equations in (1) and (5), the system has complex nonlinear characteristics.

4 Obstacle Avoidance Path Planning Based on Gauss Pseudospectral Method

In the actual obstacle avoidance task there are a variety of constraints. The traditional method based on minimum principle is difficult to obtain the optimal trajectory of intelligent vehicle obstacle avoidance. The method of numerical solution is the main research direction at present. In the research of this section, based on the kinematics and dynamics of vehicles established in section 3, a problem of time optimal intelligent vehicle obstacle avoidance control is designed by means of Gauss pseudospectral method.

4.1 Optimization Index Establishment and Constraint Analysis

For the problem of obstacle avoidance control of intelligent vehicles, the considered cost function is focused on the rapidity of obstacle avoidance, the cost function in equations (6) can be described as:

$$J = \int_{t_0}^{t_f} dt \tag{6}$$

where, t_0 and t_f respectively represent the start and terminal times of obstacle avoidance.

In the process of obstacle avoidance, the initial and terminal state of the intelligent vehicle are known and to be constrained by equation (7).

$$\begin{aligned} x(t_0) &= x_0 \\ x(t_f) &= x_f \end{aligned} \tag{7}$$

Considering that intelligent vehicle cannot accelerate infinitely to pursue the speed of obstacle avoidance, the acceleration is limited by the equation (8).

$$0 \le \dot{v}_x \le \dot{v}_{x\max} \tag{8}$$

In addition, the obstacle avoidance problem of intelligent vehicle control is based on the premise of safety. Therefore, this paper introduces the concept of safe distance, that is, the longitudinal position difference between the vehicle and the obstacle. The minimum safe distance is guaranteed as the path constraint in obstacle avoidance as shown in equation (9).

$$\begin{aligned} d_{\min} &\le d_1 = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \\ d_{\min} &\le d_2 = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2} \end{aligned} \tag{9}$$

To sum up, the general description of the obstacle avoidance control optimization problem for intelligent vehicles is as follows

$$\min J = \min \int_{t_0}^{t_f} g(x(t), u(t), t) dt \tag{10}$$

the equation and inequality constraints are satisfied.

$$\dot{x}(t) = f[x(t), u(t), t], t \in [t_0, t_f] \tag{11}$$

$$\phi[x(t_0), t_0, x(t_f), t_f] = 0 \tag{12}$$

$$C[x(t), u(t), t] \le 0 \tag{13}$$

where, $g(\cdot)$ represents the nonlinear function to be optimized. $\phi(\cdot)$ is various equation constraint functions of the system state. $C(\cdot)$ represents the system inequality constraint function. It can be seen from the above optimization problem that it is a continuous time optimization problem with nonlinear differential equation constraints, algebraic equations and inequality constraints. In this section, Gauss pseudospectrum method is used for optimal planning of obstacle avoidance path for intelligent vehicles[8].

Gauss pseudospectral method is a kind of collocation method, which use global interpolation polynomials to approximate state variables and control variables on a series LG points. The derivative of the state variables with respect to time in the kinematics and dynamics equations of the vehicle is approximated by the derivative of the interpolation polynomial. And the right function constraint of the equation is strictly satisfied at the selected coordinate points. The continuous time optimization problem is solved by discrete approximation from the following aspects[9][10].

4.2 Mapping of Time Variables

The discrete points are in interval $\tau \in [-1,1]$. However, the planned time interval of intelligent vehicle obstacle avoidance control task $[t_0, t_f]$ is not limited to $[-1,1]$, therefore, we need to map the time quantity to the interval $\tau \in [-1,1]$ as follows $t = \frac{t_f - t_0}{2}\tau + \frac{t_f + t_0}{2}$, where t_0, t_f represents the start and terminal time.

4.3 Approximation of Variables

Gauss pseudospectral method selects k LG points. $k+1$ interpolation polynomials $L_i(\tau)(i = 0, \cdots K)$ are formed. In the formula, $L_i(\tau)$ is a Lagrange interpolation basis function $L_i(\tau) = \sum_{j=0, j \ne i}^{K} \frac{\tau - \tau_j}{\tau_i - \tau_j}$. The state variable is approximated by Lagrange interpolation polynomials, $x(\tau) = X(\tau) = \sum_{i=0}^{K} L_i(\tau) U(\tau_i)$, similarly, the control variables are approximated $u(\tau) = U(\tau) = \sum_{i=0}^{K} L_i(\tau) U(\tau_i)$.

4.4 Approximation of State Derivatives

Derivating Equation $x(\tau) \approx X(\tau) = \sum_{i=0}^{K} L(\tau) x(\tau_i)$ obtain $\dot{x}(\tau_k) \approx X(\tau_k) = \sum_{i=0}^{K} \dot{L}(\tau) x(\tau_i) = \sum_{i=0}^{K} D_{ki}(\tau_k) x(\tau_i)$. Where the differential matrix $D \in R^{k(k+1)}$ can be determined offline. In this way, the differential equation constraints of the optimal control problem can be transformed into algebraic constraints $\sum_{i=0}^{K} D_{ki} X(\tau_i) - \frac{t_f - t_0}{2} f[X(\tau_k), U(\tau_k), \tau_k, t_0, t_f] = 0$.

4.5 Approximation of Terminal State Constraints

Nodes in the Gauss pseudospectral method include k collocations $(\tau_1 \cdots \tau_k)$ in $\tau \in [-1,1]$. The terminal state should satisfy the equation $X(\tau_f) = x(\tau_0) + \int_{-1}^{1} f[x(\tau), u(\tau), \tau] d\tau$. Gauss integral approximation of the terminal state is given by $X(\tau_f) = X(\tau_0) + \frac{\tau_f - \tau_0}{2} \sum_{k=-1}^{K} \omega_k f[X(\tau_k), U(\tau_k), \overline{\tau}, t_0, t_f]$.

Based on the above numerical approximation method, Gauss pseudospectral method discretizes the continuous optimal control problem and converts it into a nonlinear programming problem. The optimal trajectories of discrete state variables $(X_0, X_1 \cdots X_K)$ and control variables $(U_0, U_1 \cdots U_K)$ can be obtained by solving the nonlinear programming problem[11].

5 Simulation Research and Analysis

In this part, based on Gauss pseudospectral solution package GPOPS in simulation environment Matlab2014b, the nonlinear programming problem is solved to verify the effectiveness and performance of the design method in this paper.

5.1 Simulation Scene and Parameterser

It is assumed that vehicle A drives at 10m/s in the right lane of the two lanes (each lane is 3.5 meters wide). The obstacle C drives at a low speed of 5m/s at 50 meters directly in front of vehicle A. The obstacle avoidance strategy is to turn the steering wheel to the left lane and keep the road parallel to the original road after the obstacle avoidance. Meanwhile, 40 meters behind the left lane, there is a vehicle B drives in the same direction at the speed of 15m/s. The vehicle parameters are shown in Table 1.

Table 1: Vehicle parameters

Parameter	Value
m	1296kg
a	1.25m
b	1.32m
C_f	100700N/rad
C_r	86340N/rad
I_z	1750kgm^{-2}

The entire path planning control problem can be specifically described as follows.

Given $x = \left[x, y, \varphi, v_x, v_y, \omega, \beta, \delta_f\right]^T \in X$, find $x(\cdot), u(\cdot)$ that satisfy the constraints makes $J = t_f$. The constraints include dynamic constraints, equations (1) and equations (5) initial and terminal state constraints,

$$x_0 = [-40, -1.75, 10, 0, 0, 0, 0]$$

$$x_f = [10, 1.75, v_x, v_y, 0, 0, 0]$$

constraint of control variable, $0 \le \dot{v}_x \le 5$

path constraints, $10 \le d_1 = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$, $10 \le d_2 = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$

5.2 Simulation Results and Analysis

Through optimization calculation, a total of 42 LG nodes were selected in the solution process. The solution time is only 2.26s, optimized obstacle avoidance time is t_f 5.63s. The curves of longitudinal speed, displacement, heading angle and front wheel angle are shown in Fig. 1 to Fig. 6 shows the planning trajectory in the whole scene.

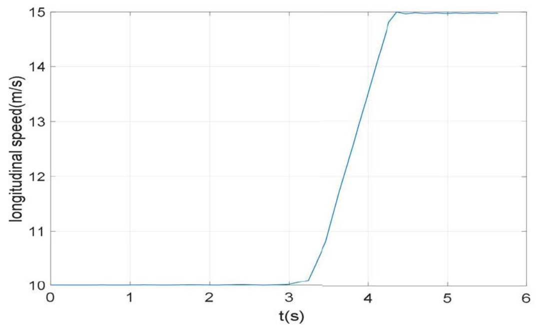

Fig. 3: Planned curves of longitudinal speed

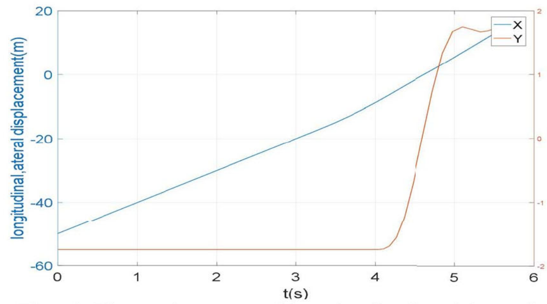

Fig. 4: Planned curves of longitudinal and lateral displacement

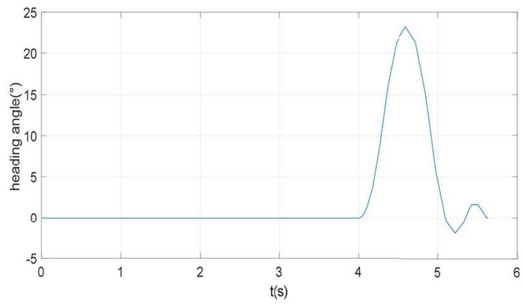

Fig. 5: Planned curves of heading angle

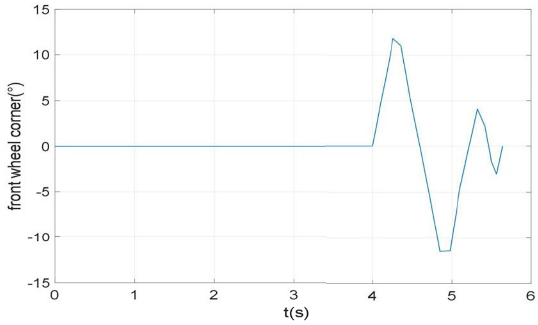

Fig. 6: Planned curves of front wheel angle

Vehicle A keeps going straight within 0-4s, and reaches the given minimum safety distance with the obstacle in front at about 4s, it began to perform lane changing actions, and the lateral displacement rose smoothly from -1.75, heading angle and the front wheel angle start to increase accordingly, the vehicle settles into its left lane and continues on, the lateral displacement is stable at 1.75, and the course angle and front wheel angle return to 0. In addition, the initial speed of the vehicle A is 10m/s, and the control variable value $\dot{v}_x$ is given 0 to 5, in this process, a accelerates gradually reached its maximum speed of 15m/s to meet the goal of optimal obstacle avoidance time. The simulation results show that the planned obstacle avoidance trajectory obtained can effectively avoid obstacles and meet the safety requirements under the conditions of various constraints, and has a fast computation speed.

6 conclusions

In order to solve the obstacle avoidance path planning of intelligent vehicles with complex constraints, an optimal trajectory planning method based on Gauss pseudospectral method is proposed. This paper establishes kinematics and dynamics models of vehicles, analyzes the state constraints and path constraints, the obstacle avoidance control problem is summarized as a continuous optimal control. The numerical solution of the optimal path is obtained by discretization. The solution environment is the Gauss pseudospectral solution package GPOPS in Matlab. Simulation results show the effectiveness of this method, which can provide reference for solving similar optimization problems. In the following research, on the basis of achieving safe obstacle avoidance, the optimization objectives such as path smoothness, ride comfort and energy saving can be further considered.

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